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Abstract

Trusses are common in many different bridge designs and analysis of the behaviour of common designs can be useful in determining which applications are best suited for each. In this project the three common designs of Howe, Pratt and Warren will be analyzed to determine where the maximum stresses occur when a moving load of 480kN moving at a constant velocity of 36km per hour or 10m/s is applied along the truss. Each truss will span 40 meters and will share material and structural properties, as well as constraints. Three methods will be used to evaluate the performance of each truss: a bridge design software package, a finite element package, and a program will be written in MATLAB to evaluate a simplified static case version of the problem. The results of the first method show that the Howe truss has greatest stresses along the top near the center. The Pratt truss has greatest stresses occurring along the top near the center and the Warren truss has the greatest stresses along the bottom near the center. The magnitude of the maximum stress is around 343MPa for each truss. Using the finite element package ANSYS the maximum stress for the Howe truss was 413MPa, the maximum nodal stress was 46MPa and the maximum deformation was 6cm. The maximum stress for the Pratt truss was 463MPa, the maximum nodal stress was 103MPa and the maximum deformation was 6.3cm. The maximum stress for the Warren truss was 490MPa, the maximum nodal stress was 109MPa and the maximum deformation was 6.7cm. Using MATLAB to write a static plane truss program the maximum nodal stress for the Howe truss was 27.5MPa and the maximum deformation was 5cm. The error between ANSYS and the bridge design software is 25%, the error between the static approximation and ANSYS is 40%. The results all seem to make physical sense. While each of these results are fairly close to each other, the differences between each of the truss designs isn't significant enough to make a recommendation to use one over the other. Each design distributes the load reasonably well as a load moves across it.

Introduction and Background

There are many different types of bridges designs in use today. Some of these include beam bridges, truss bridges, cantilever bridges, arch bridges, suspension bridges and cable-stayed bridges. Beam bridges are basic bridges supported by a series of horizontal beams crossing their span. Arch bridges use an arch to distribute loads mainly through compression across the span. Truss bridges use a series of connected members to distribute load across the span in both compression and tension. Suspension bridges use ropes or cables anchored vertically to a vertical suspender that transfers forces to towers, the major mechanism is tension. Cantilever bridges use a truss to support the bridge against a central tower. Cable-stay bridges use ropes or cables anchored directly to central towers, the major mechanism is tension. Figure 1 shows each type of bridge design.
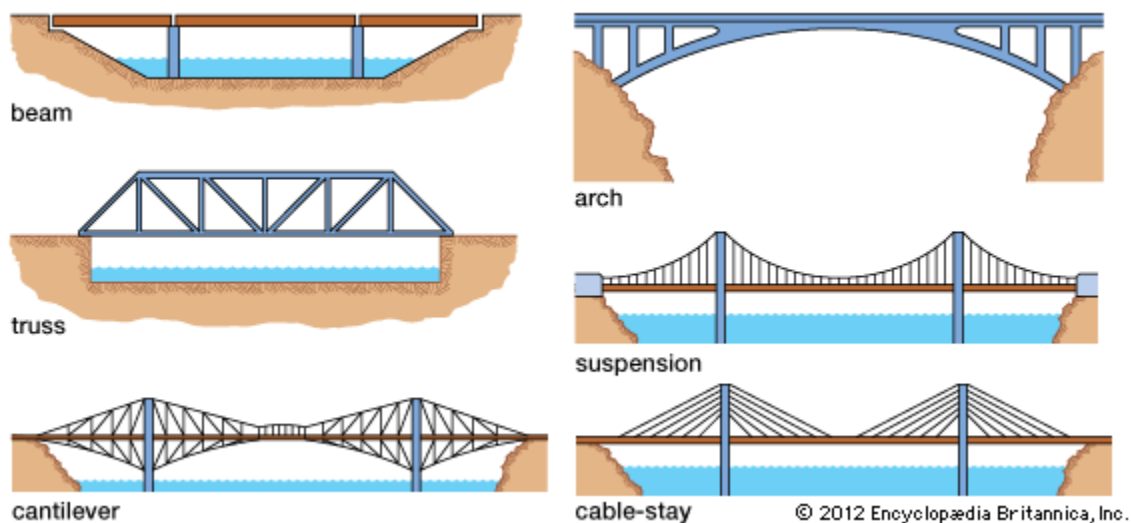

Figure 1 Types of Bridge Designs

Often for each of these designs some sort of truss system is used in addition to the main mechanism for carrying the required loads. For the truss bridge, the truss is the main mechanism for load distribution. In the cantilever bridge a series of trusses are used to distribute the forces to the central towers. In both the suspension and cable-stay bridges there is usually a truss system supporting the deck. In arch bridges there is sometimes a truss making up the arch or interfacing between the deck above the arch and the arch itself.

However there are many different truss designs in existence. Depending on the application certain truss designs will be more preferable than others. Identifying a stress distribution response of each truss to a moving load will identify the characteristic behavior of that truss. In identifying this behavior one can begin to understand which applications would be best suited to it. In this way, the best design choices can be made.

There are three main types of truss designs that other designs can be derived from. These three designs are the Howe, Pratt and Warren truss designs. The objective is to compare each of these truss designs to determine where the maximum stress concentration occurs for any single member in the truss.

Formulation

In order to make a meaningful comparison all material properties, constraints and structural characteristics must be kept common between each design. Each truss will be made out of square elements measuring 150mm × 150mm. Each truss will span 40m and be 4m high. Each truss will be loaded with a 480kN load that moves with a constant velocity of 36km/hr or 10m/s. It will take this load 4s to cross each span. Each truss will have the same material properties of quenched and tempered steel. This material has a modulus of elasticity of 200GPa. The supports or constraints for each truss are: a fixed support at one end, a frictionless or rolling support at the other end. This will be treated as a two dimensional problem and three dimensional stability will not be analyzed. The effects of fatigue and cyclic loading will also be ignored for this problem.

Three methods will be used to evaluate the behavior of each design. The first method is using the bridge design software package called West Point Bridge Designer. The second method will be to verify these results using two software packages: SolidWorks and ANSYS. SolidWorks will be used to model each design, and ANSYS will be used to perform the finite element analysis. The third method will be to write a program in MATLAB to solve a simplified version of the problem.

West Point Bridge Designer is a software package used in an annual contest for students to design an optimized bridge based on certain contest requirements that change each year. The scope of the software allows the user to build a truss bridge while controlling several aspects of the design such as beam structure and properties, span length and bridge height, beam lengths and orientation, as well as loading. The results of the simulation can be found in Appendix A and Appendix B and are discussed in the results section.

Using the bridge design software the maximum stress for the Howe truss was 343MPa at the middle top member, the maximum stress for the Pratt truss was 343MPa at the middle top member, and the maximum stress for the Warren truss was 343Mpa at the middle bottom member.

SolidWorks was used to create each of the truss structures. SolidWorks models can be seen in Appendix C. Initially an assembly of beams was created and imported into ANSYS. However due to the way mates were performed in SolidWorks, ANSYS took into account degrees of freedom that were not intended. As a result the model in SolidWorks was remade to be a single part to remove all unintended degrees of freedom. Once this was done, another issue arose in that constraints and forces needed to be applied to the bottom face of each truss but this was not possible as it was all one face. To resolve this issue, the models in SolidWorks were revised to split the bottom face into 22 parts. Two faces for constraints and 20 to apply a transient load.
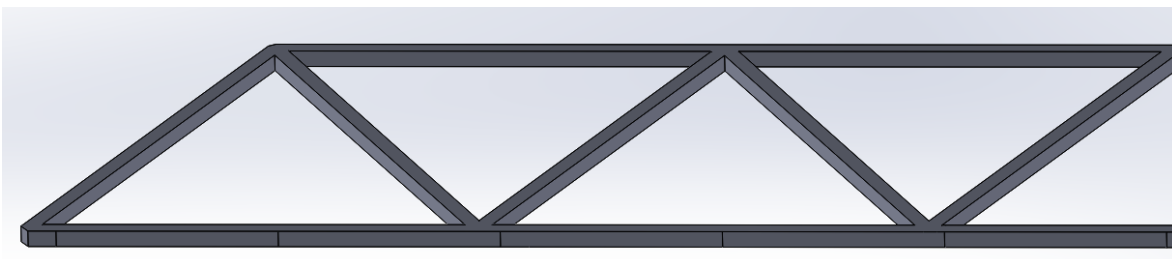

Figure 2 Splitting Bottom Face of Truss

Before solving the dynamic problem, investigation was done into how the ANSYS software package worked. Initially only a static problem could be solved in the previous status report but further investigation revealed how a dynamic problem could be solved using the transient structural analysis system. The first step was to mesh the model. For this problem experimentation with different mesh sizes was performed but the results did not change very much between fine and coarser meshes and so a coarse mesh was used because this reduced the time it took to find a solution. The coarse mesh is shown in Figure 3 for the Howe truss. As far as element types, continuum solid elements were used. This was an automatic selection by the software and did not present any problems in providing a solution so the choice was maintained. However structural elements such as shells and beams could also have been used as typically they require fewer elements compared to using solid elements. In the case of shell elements there is less stiffness and so bending is modeled efficiently. In the case of beam elements a line is used to model three dimensional continuums which results in faster computation than solid elements.
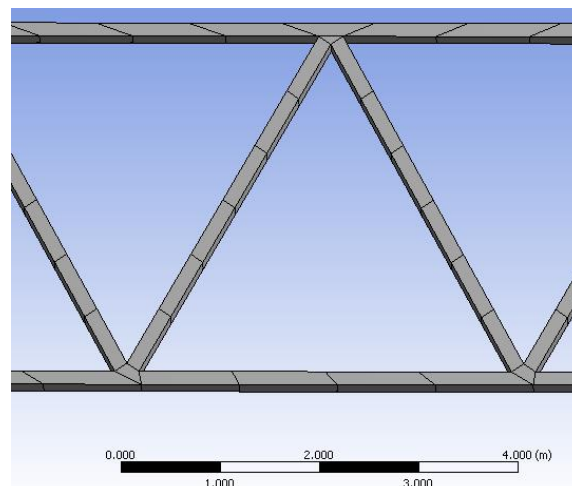

Figure 3 Howe Truss Coarse Mesh

The constraints of a fixed support on one end and a rolling support on the other were applied. In order to model a moving load a series of transient step forces of 480kN were applied to each of the bottom faces (as seen in Figure 2) of the truss. The beginning and end of these step forces overlapped each other to simulate continuous load transfer across the span of each truss. A total of 20 overlapping step forces were applied along the bottom faces of each truss. Appendix D shows this process being performed in ANSYS.

However, in attempting to solve, errors occurred. From the error logs, there appeared to be insufficient constraints for the problem. Realizing that this was a three dimensional model, a third constraint of a frictionless support was placed on the face of the truss. This third constraint was placed in order to disregard any forces or reactions in the third dimension since this is being treated as a two dimensional problem but modeled in three dimensions. This is in line with the assumption to not consider stability in the third dimension. Once this constraint was applied ANSYS was able to solve the problem. Solutions were generated for total deformation and equivalent Von-Mises stress. This process was performed for each of the Howe, Pratt and Warren trusses. Sample results for the Pratt truss can be seen in Appendix E.

The results from ANSYS indicated a maximum stress for the Howe truss of 413MPa and a maximum stress at the nodes of 46MPa. The maximum deformation for the Howe truss was 6cm. The maximum stress for

the Pratt truss was 463MPa and a maximum stress at the nodes of 103MPa. The maximum deformation for the Pratt truss was 6.3cm. The maximum stress for the Warren truss was 490MPa and a maximum stress at the nodes of 109MPa. The maximum deformation for the Warren truss was 6.7cm.

The final method involves using MATLAB to solve a simplified version of one of the trusses. The Howe truss will be solved as several static plane truss problems in order to provide an approximate representation of the dynamic case. The first step was to identify element numbers and nodes as shown in Appendix F. From this point a connectivity table was generated that also included coordinates to identify element lengths as well as angle values. Part of this table is reproduced in Appendix F.

With this information the MATLAB program was written. The modulus of elasticity, E, was set to 200GPa and the truss cross sectional area, A, was set to 0.0225m (150mm x 150mm). The parameters in the connectivity table were set up as row vectors that identified node numbers for each element as well as coordinates and angles for each element. With this information a "for loop" was entered that looped from 1 to 39 for each of the 39 elements. First lengths are calculated for each element using a function called PlaneTrussElementLength [1] that took in coordinate values and returned element lengths. Then the element stiffness matrix as in Figure 4 was generated for each element using a function called PlaneTrussElementStiffness [1] that took in values for modulus, cross sectional area, element length and angle. Then the global stiffness matrix was constructed iteratively as each of the 39 elements were processed. The function is called PlaneTrussAssemble [1] and takes in the previous global stiffness matrix, the latest element stiffness matrix and the nodes for that element. A total of 21 nodes for the system resulted in a 42x42 global matrix.

$$
\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{x2} \\ f_{y2} \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} c^2 & sc & -c^2 & -sc \\ sc & s^2 & -sc & -s^2 \\ -c^2 & -sc & c^2 & sc \\ -sc & -s^2 & sc & s^2 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \end{bmatrix} \quad \begin{array}{l} s = \sin \beta \\ c = \cos \beta \end{array}
$$

Figure 4 Truss Element Matrix [2]

With the assembled global stiffness matrix the boundary conditions were applied where no deformation in both x and y-direction at node 1 and no deformation in the y-direction at node 21 mean that the global matrix was reduced to 39x39.

A downward force of 480kN was applied at each of the following nodes one at a time 3, 5, 7, 9, 11, 13, 15, 17 and 19. The displacements were solved for using Gaussian elimination for each load application for a total of 9 sets of solutions. For each set of displacement solutions the nodal displacements for each element were identified and passed into the PlaneTrussElementStress [1] function which also took in values for modulus, element length and angle. This function returned the element stress for each truss element.

Nine sets of element stresses were obtained. The maximum stress that occurs for the Howe truss out of all of these static cases was found to be 27.5 MPa. The maximum displacement was 5.0cm. The code can be found in Appendix

Results and Discussion

The results are summarized in Table 1 Results Summary and Table 2 Error Amount:

Table 1 Results Summary

Bridge Design Software (West Point Bridge Designer)			
	Maximum Stress (MPa)	**Maximum Nodal Stress (MPa)**	**Maximum Deformation (cm)**
Howe Truss	343	-	-
Pratt Truss	343	-	-
Warren Truss	343	-	-
Finite Element Package (ANSYS)			
Howe Truss	413	46	6
Pratt Truss	463	103	6.3
Warren Truss	490	109	6.7
Program (MATLAB)			
Howe Truss	-	27.5	5.0

Table 2 Error Amount

Method	Error (%)
Bridge Design Software	25%
Finite Element Package (Baseline)	0%
Program	40%

Using the bridge design software the Pratt truss experiences maximum stresses along the top of the truss near the middle. The Warren truss experiences maximum stresses along the bottom of the truss near the middle and the Howe truss experiences maximum stresses along the top of the truss near the middle. The magnitude of the stresses is the same for each of the truss designs and this indicates that there isn't much difference between them. The error between the bridge design software and the finite element package is 25%. Some of the reasons for this might include the addition of the concrete deck which would reduce the amount of stress experienced by the truss members. In the case of the finite element package the truss takes all of the load.

The results from ANSYS are used as a baseline to compare the rest of the results against because they are expected to be the most accurate. The results from the finite element package show increased maximum

stress values compared to the bridge design software. Although the magnitude of the stresses is still close to those given by the software. In a similar manner to the bridge design software these all occur at the middle of the structure. One major difference between the two analyses is that a moving load was applied directly to the bottom surface of the truss. This resulted in large stresses and deformations in the locations between nodes along the bottom of the beam. It is possible that this can also account for the difference in results. One interesting behaviour noticed in the transient problem is that vibration occurs in the truss once the load has moved off the far end, however this vibration is damped out quickly.

The deformation was also solved for and the values for deformation make physical sense. The largest deformation is just under 7cm which is reasonable for a truss spanning 40m. In addition values for the maximum stresses near the nodes were identified. These values along with the values for deformation can be checked against the static approximation performed using MATLAB.

It was found that the results from the MATLAB program were reasonably close to those found using ANSYS. Especially the deformation which was within 1cm of the results from ANSYS. However the maximum stress in any element was lower than that for ANSYS by 40%. This can be attributed to the nature of the approximation being a static case versus the transient case in ANSYS. Also the truss was approximated as truss elements in the program versus more finely discretized elements in the finite element package. The coarser elements used in the MATLAB program would be expected to give lower values since they would be stiffer. Another disadvantage of this approximation is that it can only apply loads at nodes instead of continuously across the truss. The approximation was only performed for the Howe truss as a sample. It would be expected that the results would be similar for the Pratt and Warren trusses

Conclusion and Future Work

The results of this analysis seem to make physical sense and the animations seen in both ANSYS and the bridge design software are as expected. All of the results are within an order of magnitude of each other and this is encouraging. Each of the three methods have helped verify each other. From the results there isn't enough of difference between each of the trusses in order to recommend one design over the other. Future work in this area would be to increase the complexity of the ANSYS model to more closely model real world situations which include wind loading and a body force due to gravity. The model could also be expanded to three dimensions. Furthermore the mass that was moved across the truss could be instead be two points at once to approximate the front and rear axle of a vehicle. Also more complex truss configurations could be evaluated.

References

[1] P. Kattan, MATLAB Guide to Finite Elements, New York: Springer, 2008.

[2] D. L. Logan, A First Course in the Finite Element Method, Fourth Edition, Toronto: Nelson, 2007.

Howe Truss

Pratt Truss

Warren Truss

40.00m

40.00m

40.00m

4.00m

4.00m

4.00m
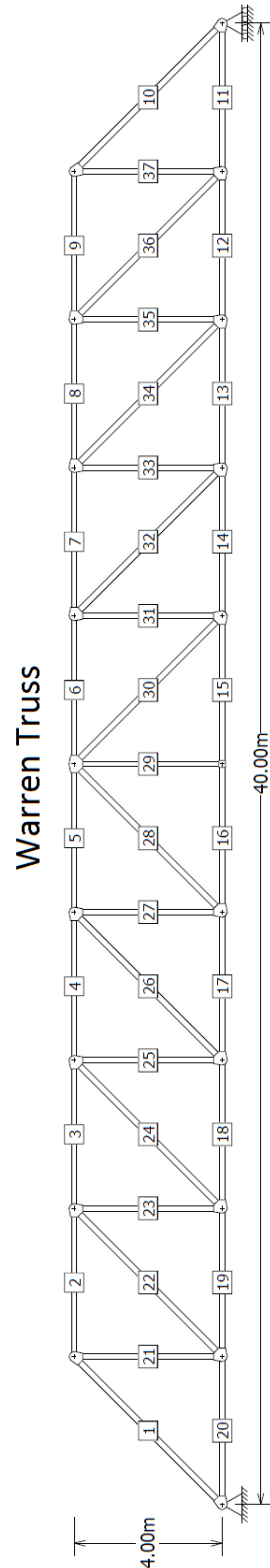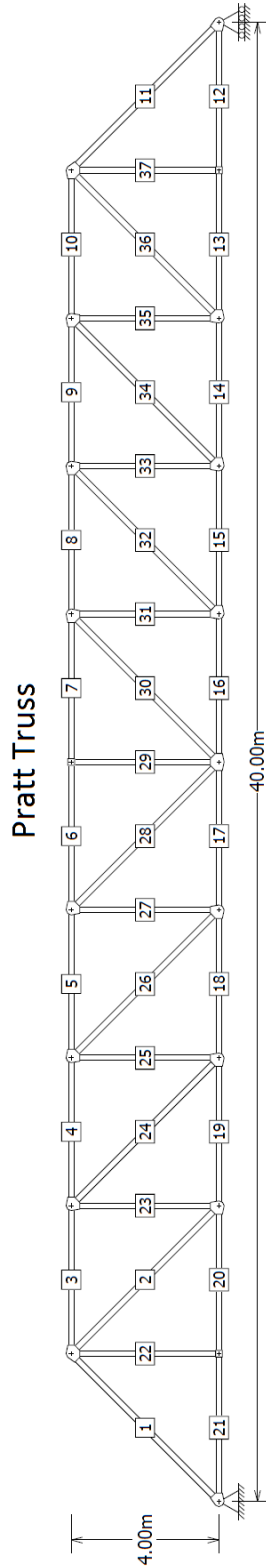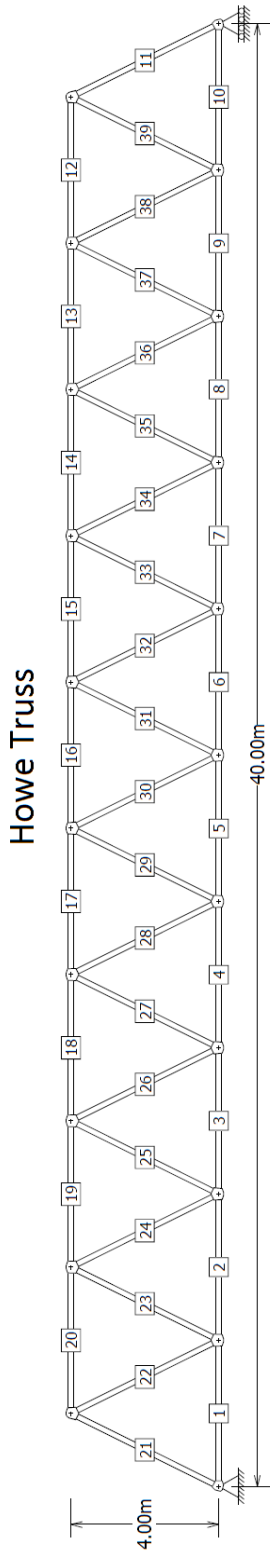
Appendix B – West Point Bridge Designer Truss Analysis

Howe Truss

| #	Material Type	Cross Section	Size (mm)	Length (m)	Compression Force	Compression Strength	Compression Status	Tension Force	Tension Strength	Tension Status
1	QTS	Bar	150	5.66	1776.42	2061.05	OK	0.00	10366.88	OK
2	QTS	Bar	150	4.00	1256.12	4109.69	OK	0.00	10366.88	OK
3	QTS	Bar	150	4.00	2230.26	4109.69	OK	0.00	10366.88	OK
4	QTS	Bar	150	4.00	2918.09	4109.69	OK	0.00	10366.88	OK
5	QTS	Bar	150	4.00	3319.62	4109.69	OK	0.00	10366.88	OK
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27	QTS	Bar	150	4.00	0.00	4109.69	OK	549.99	10366.88	OK
28	QTS	Bar	150	5.66	478.94	2061.05	OK	153.04	10366.88	OK
29	QTS	Bar	150	4.00	0.00	4109.69	OK	428.65	10366.88	OK
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Pratt Truss

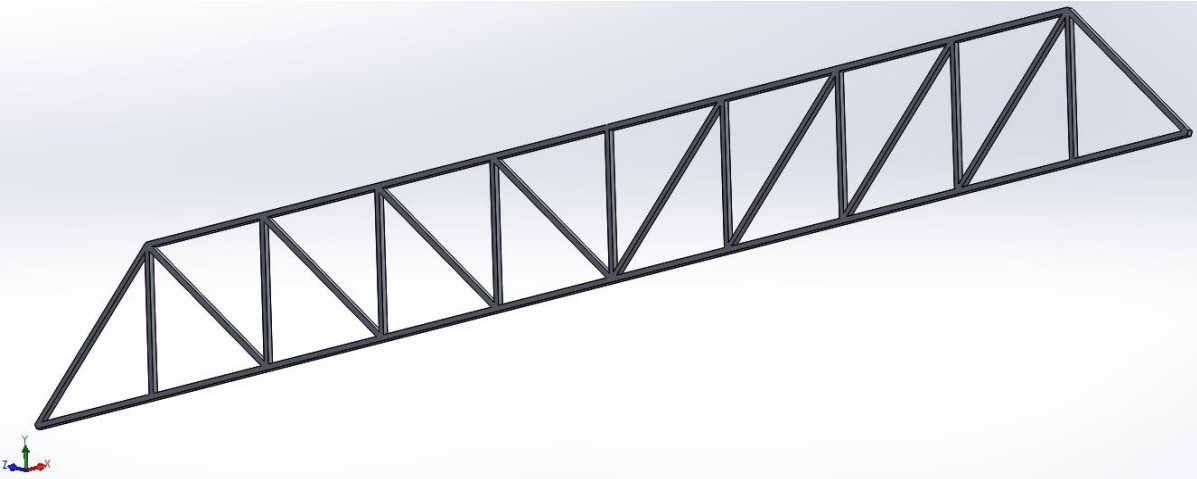| #	Material Type	Cross Section	Size (mm)	Length (m)	Compression Force	Compression Strength	Compression Status	Tension Force	Tension Strength	Tension Status
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Warren Truss

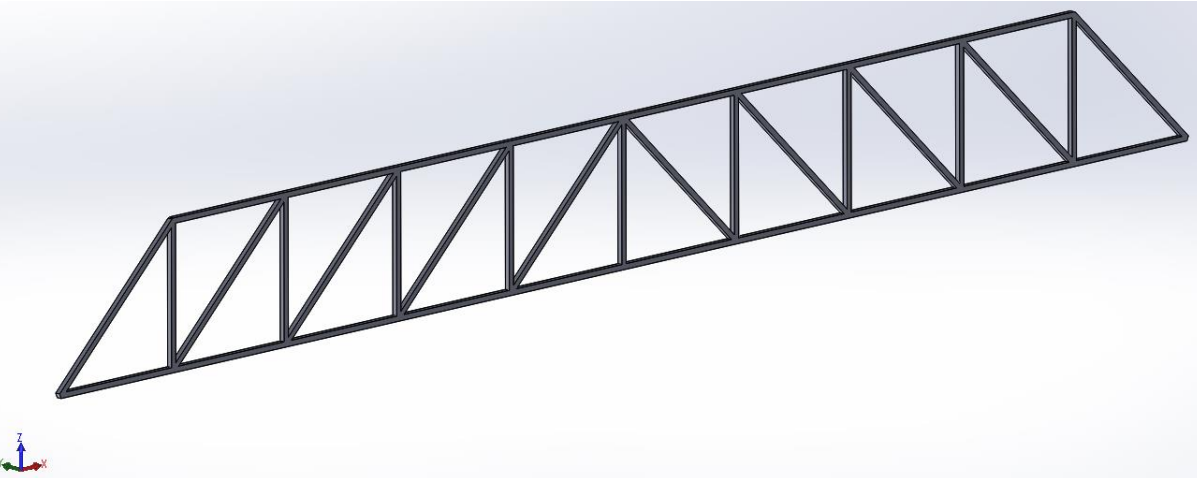| #	Material Type	Cross Section	Size (mm)	Length (m)	Compression Force	Compression Strength	Compression Status	Tension Force	Tension Strength	Tension Status
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17	QTS	Bar	150	4.00	3303.27	4109.69	OK	0.00	10366.88	OK
18	QTS	Bar	150	4.00	2904.06	4109.69	OK	0.00	10366.88	OK
19	QTS	Bar	150	4.00	2220.08	4109.69	OK	0.00	10366.88	OK
20	QTS	Bar	150	4.00	1251.34	4109.69	OK	0.00	10366.88	OK
21	QTS	Bar	150	4.47	1406.87	3297.68	OK	0.00	10366.88	OK
22	QTS	Bar	150	4.47	0.00	3297.68	OK	1391.21	10366.88	OK
23	QTS	Bar	150	4.47	1155.79	3297.68	OK	0.00	10366.88	OK
24	QTS	Bar	150	4.47	0.00	3297.68	OK	1135.29	10366.88	OK
25	QTS	Bar	150	4.47	899.87	3297.68	OK	0.00	10366.88	OK
26	QTS	Bar	150	4.47	0.00	3297.68	OK	879.36	10366.88	OK
27	QTS	Bar	150	4.47	643.95	3297.68	OK	0.00	10366.88	OK
28	QTS	Bar	150	4.47	0.00	3297.68	OK	623.44	10366.88	OK
29	QTS	Bar	150	4.47	388.03	3297.68	OK	111.60	10366.88	OK
30	QTS	Bar	150	4.47	132.11	3297.68	OK	367.52	10366.88	OK
31	QTS	Bar	150	4.47	132.11	3297.68	OK	367.52	10366.88	OK
32	QTS	Bar	150	4.47	388.03	3297.68	OK	111.60	10366.88	OK
33	QTS	Bar	150	4.47	0.00	3297.68	OK	623.44	10366.88	OK
34	QTS	Bar	150	4.47	643.95	3297.68	OK	0.00	10366.88	OK
35	QTS	Bar	150	4.47	0.00	3297.68	OK	879.36	10366.88	OK
36	QTS	Bar	150	4.47	899.87	3297.68	OK	0.00	10366.88	OK
37	QTS	Bar	150	4.47	0.00	3297.68	OK	1135.29	10366.88	OK
38	QTS	Bar	150	4.47	1155.79	3297.68	OK	0.00	10366.88	OK
39	QTS	Bar	150	4.47	0.00	3297.68	OK	1391.21	10366.88	OK

Appendix C – SolidWorks Truss Models
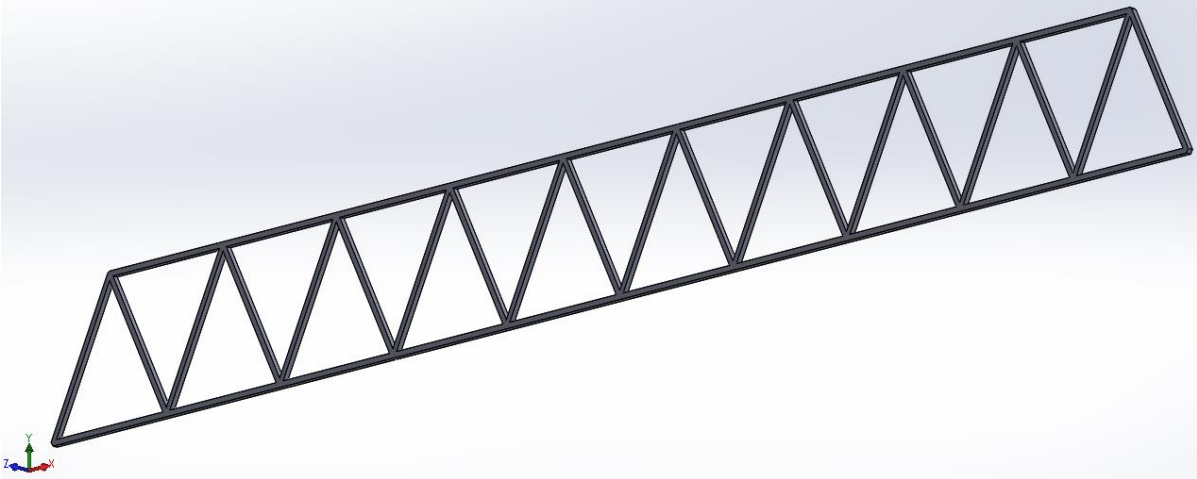
Howe Truss

Pratt Truss

Warren Truss

Appendix D – Application of Transient Step Forces in ANSYS

Appendix E – Sample Results ANSYS Pratt Truss
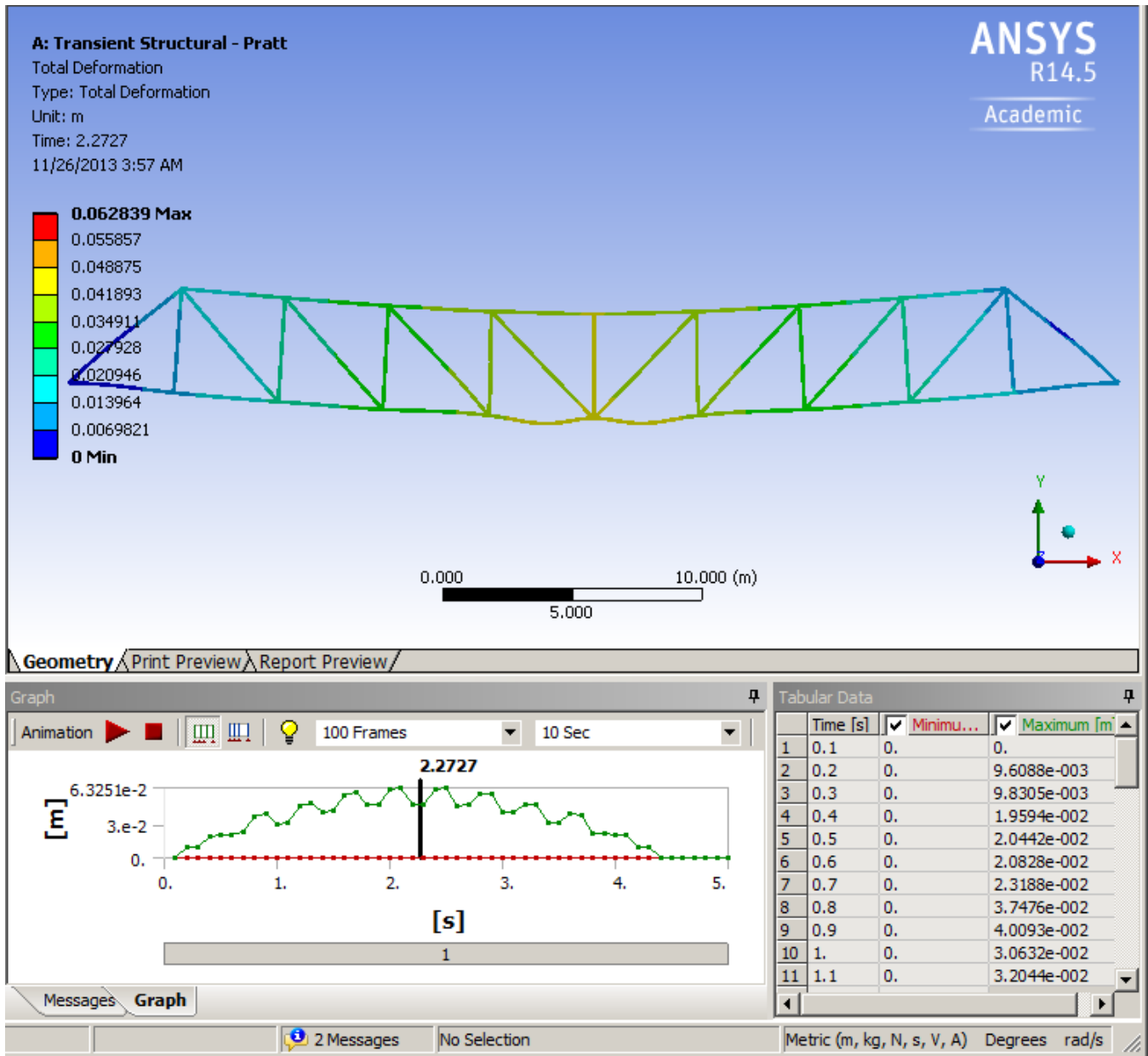
Total Deformation
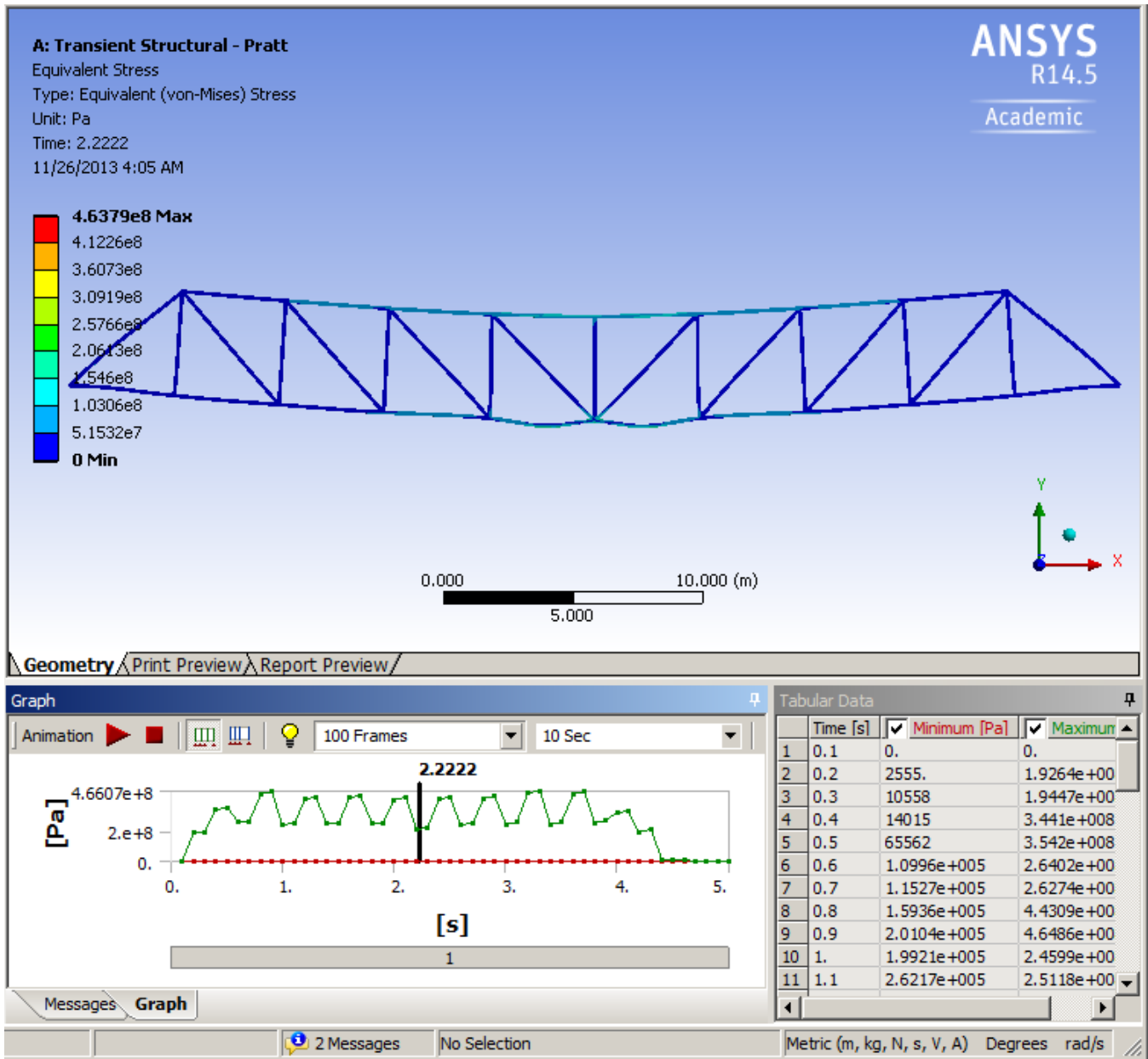
Equivalent von-Mises Stress

Appendix F – Static Approximation using MATLAB

Howe Truss
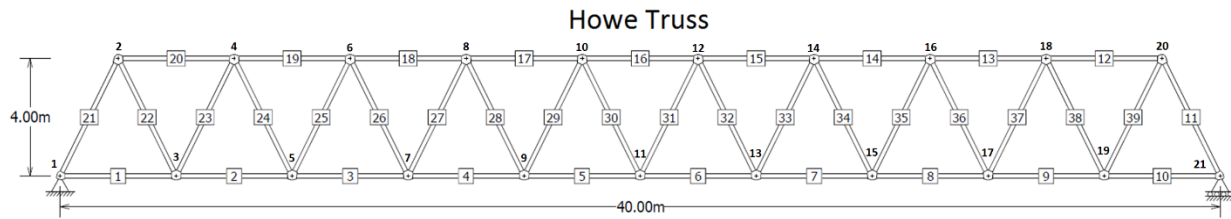
Element and Node Numbering

Element	Node i	Node j	x_1	y_1	x_2	y_2	θ
1	1	3	0	0	4	0	0
2	3	5	4	0	8	0	0
3	5	7	8	0	12	0	0
4	7	9	12	0	16	0	0
5	9	11	16	0	20	0	0
6	11	13	20	0	24	0	0
7	13	15	24	0	28	0	0
8	15	17	28	0	32	0	0
...	...	...	...	...	...	...	...
32	12	13	22	4	24	0	233.13
33	13	14	24	0	26	4	63.43
34	14	15	26	4	28	0	233.13
35	15	16	28	0	30	4	63.43
36	16	17	30	4	32	0	233.13
37	17	18	32	0	34	4	63.43
38	18	19	34	4	36	0	233.13
39	19	20	36	0	38	4	63.43

Appendix G – MATLAB Code

```matlab
L=[];
k=[];
K=zeros(42,42);
sigma=[];

E=2e11;
A=0.0225;

Nodei=[1,3,5,7,9,11,13,15,17,19,20,18,16,14,12,10,8,6,4,2,1,2,3,4,5,...
    6,7,8,9,10,11,12,13,14,15,16,17,18,19];
Nodej=[3,5,7,9,11,13,15,17,19,21,21,20,18,16,14,12,10,8,6,4,2,3,4,5,...
    6,7,8,9,10,11,12,13,14,15,16,17,18,19,20];
x1=[0,4,8,12,16,20,24,28,32,36,38,34,30,26,22,18,14,10,6,2,0,2,4,6,8,...
    10,12,14,16,18,20,22,24,26,28,30,32,34,36];
y1=[0,0,0,0,0,0,0,0,0,0,4,4,4,4,4,4,4,4,4,0,4,0,4,0,4,0,4,0,4,0,4,0,...
    4,0,4,0,4,0,4,0];
x2=[4,8,12,16,20,24,28,32,36,40,40,38,34,30,26,22,18,14,10,6,2,4,6,8,...
    10,12,14,16,18,20,22,24,26,28,30,32,34,36,38];
y2=[0,0,0,0,0,0,0,0,0,0,4,4,4,4,4,4,4,4,4,4,0,4,0,4,0,4,0,4,0,4,0,4,...
    0,4,0,4,0,4];
theta=[0,0,0,0,0,0,0,0,0,0,233.13,0,0,0,0,0,0,0,0,0,63.43,233.13,...
    63.43,233.13,63.43,233.13,63.43,233.13,63.43,233.13,63.43,233.13,...
    63.43,233.13,63.43,233.13,63.43,233.13,63.43];

for i=1:39
    L(i)=PlaneTrussElementLength(x1(i),y1(i),x2(i),y2(i));
    k=PlaneTrussElementStiffness(E,A,L(i),theta(i));
    K=PlaneTrussAssemble(K,k,Nodei(i),Nodej(i));
end

K;
NewK=K(3:41,3:41);

f=[0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;-450000;0;0;0;0;0;0;...
    0;0;0;0;0;0;0;0;0;0];
u=NewK\f;

U=[0;0;u(1:39);0];

F=K*U;

for i=1:39
    U_El=[U(2*Nodei(i)-1);U(2*Nodei(i));U(2*Nodej(i)-1);U(2*Nodej(i))];
    sigma(i)=PlaneTrussElementStress(E,L(i),theta(i),U_El);
end

transpose(sigma);
```